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Neutrino diffraction induced by light-cone singularity and small mass

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Abstract

The many-body wave function of a pion and its decay products shows that the probability of detecting a neutrino possesses a unique finite size correction. The detection rate at a finite distance L is expressed as $\Gamma_0 + \tilde{g}(\omega_\nu \mathrm{L}/c)\Gamma_1$, where $\omega_\nu = m_\nu^2 c^4 / 2E_\nu \hbar$ and c is the light velocity. Γ_0 is a constant that is computed with the standard S-matrix of plane waves and the second term is a finite size correction that is computed with wave packets. The value of $\tilde{g}(x)$ decreases rapidly with x and vanishes in charged leptons, but is finite in neutrinos at a macroscopic L. The finite size correction reveals a diffraction pattern of a single neutrino and can be computed rigorously with the light-cone singularity of a system consisting of a pion and a muon. We predict that the neutrino diffraction would be observed at near-detector regions of ground experiments and that it could be used for the experimental determination of the neutrino mass.

1 Neutrino interference. Interference phenomena of photons, electrons, neutrons and other heavy elements are useful for testing quantum mechanics and other basic principles. In this paper, we show that a neutrino produced by pion decay displays an interference phenomenon that could provide an absolute value for the neutrino mass from its unique interference pattern.

A neutrino interacts extremely weakly with matters; hence, a quantum-mechanical wave function that expresses a system consisting of a pion and a charged lepton, and a neutrino wave, become highly correlated. Consequently, the detection probability of the neutrino becomes dependent on a distance between the initial pion and the neutrino detector, which we call a finite size correction. An S-matrix $S[\mathrm{T}]$ defined according to the boundary condition at the finite size, where T stands for a time interval, does not satisfy various relations [1] of an S-matrix of the infinite size $S[\infty]$. Wave packets, localized around center positions, satisfy the asymptotic boundary conditions [2, 3] even in this case and are suitable for studying the finite size corrections of transition probabilities. Accordingly we compute the transition amplitude with the S-matrix of the finite time interval, $S[\mathrm{T}]$, defined using wave packets.

We find that the detection rate is composed of a constant and a finite size correction. The former agrees with a standard value obtained by an S-matrix of plane waves, while the latter is a new term that cannot be computed by it but by $S[\mathrm{T}]$ and has an origin in a diffraction. Waves accumulating at the light velocity form a light-cone singularity and exhibit a neutrino diffraction phenomenon, which makes the finite size correction long range. The diffraction pattern is determined by the difference of angular velocities, $\omega_\nu = \omega_\nu^E - \omega_\nu^{dB}$, where $\omega_\nu^E = E_\nu/\hbar$ and $\omega_\nu^{dB} = c|\vec{p}_\nu|/\hbar$. ω_ν becomes an extremely small value $m_\nu^2 c^4/2E_\nu\hbar$ owing to unique neutrino features [4–6]. Consequently, the diffraction term becomes finite in a macroscopic spatial region $r \leq \frac{2\pi E_\nu \hbar c}{m_\nu^2 c^4}$ and affects experiments in a mass-dependent manner at near-detector regions.

A neutrino wave packet [7–9] expresses a nucleon wave function in a nucleus with which the neutrino interacts and is well localized [10–16]. Mass-squared differences δm_ν^2 are negligible [4], thus, we study a situation in which the mass-squared average $\bar{m}_\nu^2$ satisfies, $\bar{m}_\nu^2 \gg \delta m_\nu^2$, and present one flavor case. Extensions to general cases are straightforward.

2 Position-dependent probability. The position-dependent probability of detecting a neutrino is computed with $S[\mathrm{T}]$, which has two components owing to the algebra of [1]

and is expressed as

$$\langle\beta|S[\mathrm{T}]|\alpha\rangle = \langle\beta|S^{(n)}[\mathrm{T}]|\alpha\rangle + \langle\beta|S^{(d)}[\mathrm{T}]|\alpha\rangle, \tag{1}$$

where the first term satisfies $E_\beta = E_\alpha$ and the second term satisfies $E_\beta \neq E_\alpha$. The process by which a pion is prepared at one time and a neutrino is detected at another is described by the time-dependent Schrödinger equation and the position-dependent amplitude $T = \int d^4x\, \langle \mu, \nu|H_w(x)|\pi\rangle$ between a pion prepared at T_π, a wave packet of a neutrino at $(\mathrm{T}_\nu, \vec{\mathrm{X}}_\nu)$ and an unmeasured-muon. They are expressed as $|\pi\rangle = |\vec{p}_\pi, \mathrm{T}_\pi\rangle$, $|\mu,\nu\rangle = |\mu, \vec{p}_\mu; \nu, \vec{p}_\nu, \vec{\mathrm{X}}_\nu, \mathrm{T}_\nu\rangle$. T is written with the pion field and Dirac spinors as

$$T = \int d^4x d\vec{k}_\nu\, N\langle 0|\varphi_\pi(x)|\pi\rangle \bar{u}(\vec{p}_\mu)(1-\gamma_5)\nu(\vec{k}_\nu) \\ \times e^{ip_\mu\cdot x + ik_\nu\cdot(x-\mathrm{X}_\nu) - \frac{\sigma_\nu}{2}(\vec{k}_\nu - \vec{p}_\nu)^2},$$

where $N = igm_\mu\,(\sigma_\nu/\pi)^{\frac{4}{3}}\,(m_\mu m_\nu/E_\mu E_\nu)^{\frac{1}{2}}$, and t is integrated in the region $\mathrm{T}_\pi \leq t$. The value for $\vec{k}_\nu$ must be integrated in the whole region in order to represent the localized wave packet in coordinate variables. For simplicity, a Gaussian form is assumed in most places. We will show that the finite size correction has a universal property that is common to general wave packets. The size of the wave packet, σ_ν, is estimated later. The amplitude T depends upon the coordinates $(\mathrm{T}_\nu, \vec{\mathrm{X}}_\nu)$ and T_π and the probability depends on $\mathrm{T} = \mathrm{T}_\nu - \mathrm{T}_\pi$.

Integrating $\vec{k}_\nu$, we obtain a Gaussian function of $\vec{x}$, which vanishes at large $|\vec{x}|$ and satisfies the asymptotic boundary condition. We express the square of the modulus of T with a correlation function. Because the order of integrations is interchangeable, the muon momentum is integrated first for a fixed x_i. Then, after the spin summations are made, the transition probability is written with the correlation function $\Delta(x_1, x_2)$,

$$\Delta_{\pi,\mu}(\delta x) = \frac{1}{(2\pi)^3}\int \frac{d\vec{p}_\mu}{E(\vec{p}_\mu)}(p_\mu\cdot p_\nu)e^{-i(p_\pi - p_\mu)\cdot\delta x}. \tag{2}$$

as,

$$P = \frac{C}{E_\nu}\int d^4x_1 d^4x_2 e^{-\frac{1}{2\sigma_\nu}\sum_i(\vec{x}_i - \vec{x}_i^{\,0})^2}\Delta_{\pi,\mu}(\delta x)e^{i\phi(\delta x)}, \tag{3}$$

where $C = g^2m_\mu^2\,(4\pi/\sigma_\nu)^{\frac{3}{2}}\,V^{-1}$, V is the normalization volume for the initial pion, $\vec{x}_i^{\,0} = \vec{\mathrm{X}}_\nu + \vec{v}_\nu(t_i - \mathrm{T}_\nu)$, $\delta x = x_1 - x_2$, $\phi(\delta x) = p_\nu\cdot\delta x$. In Eq. (2), the muon momentum is integrated in the whole positive energy region in order for Eq. (3) to agree with the original probability.

3 Light-cone singularity. $\Delta_{\pi,\mu}(\delta x)$ has two components. One is singular and extended in large $|\delta\vec{x}|$ and the other is regular and localized in small $|\delta\vec{x}|$. A superposition of infinite waves of the same phase leads to the first component. This term is easily extracted by an expression of the integral in the four-dimensional form with the new variable $q = p_\mu - p_\pi$ that is conjugate to δx. Then $\Delta_{\pi,\mu}(\delta x)$ is decomposed into the integrals of the regions $0 \leq q^0$ and $-p_\pi^0 \leq q^0 \leq 0$. The integral in the region $0 \leq q^0$ is expressed as, $(p_\pi\cdot p_\nu - ip_\nu\cdot(\frac{\partial}{\partial\delta x}))\tilde{I}_1$, where

$$\tilde{I}_1 = \int d^4q\, \frac{\theta(q^0)}{4\pi^4}\mathrm{Im}\left[\frac{1}{q^2 + 2p_\pi\cdot q + \tilde{m}^2 - i\epsilon}\right] e^{iq\cdot\delta x},$$

and $\tilde{m}^2 = m_\pi^2 - m_\mu^2$. The integrand of $\tilde{I}_1$ is expanded in $p_\pi\cdot q$ and the integration leads to the light-cone singularity [17], $\delta(\delta x^2)$, and less singular and regular terms that are described with Bessel functions. The integral from the region $-p_\pi^0 \leq q^0 \leq 0$, I_2, is written with the momentum $\tilde{q} = q + p_\pi$ and has no singularity. Thus the correlation function, $\Delta_{\pi,\mu}(\delta x)$, is expressed as

$$\begin{aligned}\Delta_{\pi,\mu}(\delta x) = 2i&\left\{p_\pi\cdot p_\nu - ip_\nu\cdot\left(\frac{\partial}{\partial\delta x}\right)\right\}\\ \times&\left[D_{\tilde{m}}\left(-i\frac{\partial}{\partial\delta x}\right)\left(\frac{\epsilon(\delta t)}{4\pi}\delta(\lambda) + f_{short}\right) + I_2\right],\end{aligned} \tag{4}$$

where $\lambda = (\delta x)^2$, $D_{\tilde{m}}(-i\frac{\partial}{\partial\delta x}) = \sum_l (1/l!)\big(2p_\pi\cdot(-i\frac{\partial}{\partial\delta x})\,\frac{\partial}{\partial\tilde{m}^2}\big)^l$, $f_{short} = -\frac{i\tilde{m}^2}{8\pi\xi}\,\theta(-\lambda)\,\{N_1(\xi) - i\,\epsilon(\delta t)\,J_1(\xi)\} - \frac{i\tilde{m}^2}{4\pi^2\xi}\,\theta(\lambda)\,K_1(\xi)$, $\xi = \tilde{m}\sqrt{\lambda}$, N_1, J_1, and K_1 are Bessel functions. f_{short} has a singularity of the form $1/\lambda$ around $\lambda = 0$ and decrease as $e^{-\tilde{m}\sqrt{|\lambda|}}$ or oscillates as $e^{i\tilde{m}\sqrt{|\lambda|}}$ at large $|\lambda|$. The condition for the convergence of the series will be studied later.

Integration of spatial coordinates. Next, Eq.(4) is substituted in Eq.(3) and $\vec{x}_1$ and $\vec{x}_2$ are integrated. The first term, $J_{\delta(\lambda)}$, is derived from the most singular term, $\frac{\epsilon(\delta t)}{4\pi}\delta(\lambda)$, and computed with the center coordinate $X^\mu = (x_1^\mu + x_2^\mu)/2$ and the relative coordinate $\vec{r} = \vec{x}_1 - \vec{x}_2$. Integrating these variables, we have $J_{\delta(\lambda)}$ as

$$J_{\delta(\lambda)} = C_{\delta(\lambda)}\frac{\epsilon(\delta t)}{|\delta t|}e^{i\bar{\phi}_c(\delta t) - \frac{m_\nu^4 c^8}{16\sigma_\nu E_\nu^4}\delta t^2}, \tag{5}$$

where $C_{\delta(\lambda)} = (\sigma_\nu\pi)^{\frac{3}{2}}\sigma_\nu/2$ and $\bar{\phi}_c(\delta t) = \omega_\nu\delta t = \delta t\ m_\nu^2c^4/2E_\nu$. We note that the phase $\phi(\delta x)$ of Eq. (3) became the small phase $\bar{\phi}_c(\delta t)$ of Eq. (5) at the light cone $\lambda = 0$. The next singular term is from $1/\lambda$ in $\Delta_{\pi,\mu}$, and becomes $J_{\delta(\lambda)}/\sqrt{\pi\sigma_\nu p_\nu^2}$, which is much smaller than $J_{\delta(\lambda)}$ in the present parameter region. The magnitude is inversely proportional to $|\delta t|$ and is

independent of $\tilde{m}^2$. This behavior is satisfied in general forms of the wave packets and the following theorem is proved.

Theorem: wave packets of spreading non-Gaussian forms In general cases, the leading term $J_{\delta(\lambda)}$ of the wave packets of spreading non-Gaussian forms has the phase and the magnitude of the form Eq. (5). The phase is given as a sum of $\bar{\phi}_c(\delta t)$ and a small correction that becomes $O(1/E_\nu)$ in general and $O(1/E_\nu^2)$ in time inversion invariant wave packets.

(Proof.) The Gaussian function of Eq.(3) is replaced with the wave function $\psi(\vec{x}-\vec{v}t)$

$$\psi(\vec{x}-\vec{v}t) = \int dk_l d\vec{k}_T \, e^{ik_l(x_l - v_\nu t) + i\vec{k}_T\cdot\vec{x}_T + iC_{ij}k_T^i k_T^j t} \times \psi_k(k_l,\ \vec{k}_T),\ C_{ij} = \delta_{ij}/2E, \tag{6}$$

where the last term in the exponent is from an expansion of $E(\vec{p}+\vec{k})$ and makes the wave packet spread with time and $\psi_k(k_l,\ \vec{k}_T)$ is a wave function in the momentum. Since the coefficient C_{ij} in the longitudinal direction is negligible for the neutrino, it is neglected. The center coordinate $\vec{X}$ is easily integrated in Eq. (3), and the product of $\psi(\vec{x}-\vec{v}t)$ and its complex conjugate is reduced to $\tilde{\psi}(r_l - v_\nu\delta t, \vec{r}_T)$ and is written as

$$\tilde{\psi}(r_l - v_\nu\delta t, \vec{r}_T) = \int dk_l d\vec{k}_T e^{ik_l(r_l - v_\nu\delta t)} \times e^{i\vec{k}_T\cdot\vec{r}_T + i(\vec{k}_T^2/2E)\delta t}|\psi_k(k_l,\vec{k}_T)|^2. \tag{7}$$

Then the leading term $J_{\delta(\lambda)} = \int d\vec{r}\, e^{i\phi(\delta x)}\tilde{\psi}(\vec{r}-\vec{v}_\nu\delta t)\frac{\epsilon(\delta t)}{4\pi}\delta(\lambda)$, becomes $J_{\delta(\lambda)} = \pi e^{-\frac{1}{2}}w_0(1+\gamma)\frac{\epsilon(\delta t)}{2\delta t}e^{i\bar{\phi}_c(\delta t)(1+\delta)}$ at a large $|\delta t|$ region, where $w_0 = \int dk_l|\psi_k(k_l,0)|^2$, $\delta = d_1/E + d_2/2E^2$, $\gamma = d_1/2E + (d_2/2!)\times(1/2E)^2 - (1-v_\nu)^2\delta t^2$, $d_1 = {w_0}^{-1}\int dk_l k_l|\psi_k(k_l,0)|^2$ and $d_2 = {w_0}^{-1}\int dk_l k_l^2|\psi_k(k_l,0)|^2$. In a wave packet of time reversal invariance, $|w(k_l,0)|^2$ is an even function of k_l, hence $d_1 = 0$. The correction terms become negligible for $E_\nu \geq 2-300$ [MeV]. **(Q.E.D.)** Thus the theorem is proved. This theorem demonstrates that the neutrino fluxes for any collision are modified with the diffraction component.

Next we evaluate the integrals of the regular terms of $\Delta_{\pi,\mu}$. These terms are oscillating or decreasing rapidly with λ and those of $\vec{r}\approx 0$ contribute to this. Hence, the spreading effect is negligible, and the Gaussian approximation is good.

The first term, $\tilde{L}_1$, is from f_{short} in Eq.(4). In the space-like region $\lambda < 0$, the asymptotic expressions of the Bessel functions at large $|\delta t|$ give $\tilde{L}_1 = C_1|\delta t|^{-\frac{3}{4}}e^{i(E_\nu - p_\nu v_\nu)\delta t - \sigma_\nu p_\nu^2 + i\tilde{m}\sqrt{|2v_\nu\sigma_\nu p_\nu\delta t|}}$,

where $C_1 = i\frac{\sigma_\nu}{4}\left(\frac{\sigma_\nu \tilde{m}}{2}\right)^{\frac{1}{2}}(4v_\nu\sigma_\nu p_\nu)^{-\frac{3}{4}}$. On the other hand, in the time-like region $\lambda > 0$, $\tilde{L}_1$ decreases with time as $e^{-\tilde{m}b_1\sqrt{|\delta t|}}$. The second term, $\tilde{L}_2$, is from I_2, which is approximately the integral of $e^{-i\left(E_\pi - E_\nu - \sqrt{q^2+m_\mu^2}\right)\delta t}$ in $\vec{q}$ in a range $1/\sqrt{\sigma_\nu}$. Thus $\tilde{L}_2$ is a steep decreasing function of δt.

4 Time-dependent probability. Finally we integrate t_1, t_2 in,

$$P = N_1 \int dt_1 dt_2 \left[\frac{\epsilon(\delta t)}{|\delta t|}e^{i\bar{\phi}_c(\delta t)} + 2D_{\tilde{m}}(p_\nu)\frac{\tilde{L}_1}{\sigma_\nu} - \frac{2i}{\pi}\left(\frac{\sigma_\nu}{\pi}\right)^{\frac{1}{2}}\tilde{L}_2\right], \tag{8}$$

where $N_1 = ig^2 m_\mu^2 \pi^3 \sigma_\nu (8p_\pi \cdot p_\nu / E_\nu)V^{-1}$. In the above equation, and most other places, the low neutrino mass is neglected compared to $\tilde{m}^2$, $p_\pi \cdot p_\nu$ and σ_ν, except the slow phase $\bar{\phi}_c(\delta t)$. The first term in Eq. (8) oscillates slowly with time δt and the remaining terms oscillate or decrease rapidly. They are clearly separated.

Here we study the series $\sum_n(-2p_\pi \cdot p_\nu)^n \frac{1}{n!}\left(\frac{\partial}{\partial \tilde{m}^2}\right)^n \tilde{L}_1$, in Eq. (8). This series converges when the most diverging term, $S_1 = \sum_n(-2p_\pi \cdot p_\nu)^n \frac{1}{n!}\left(\frac{\partial}{\partial \tilde{m}^2}\right)^n (\tilde{m}^2)^{\frac{1}{4}} = \sum_n \left(\frac{2p_\pi p_\nu}{\tilde{m}^2}\right)^n n^{-\frac{5}{4}}(\tilde{m})^{\frac{1}{2}}$ becomes finite. This is clearly seen in $2p_\pi \cdot p_\nu < \tilde{m}^2$. At $2p_\pi \cdot p_\nu = \tilde{m}^2$, S_1 becomes finite, and the value is expressed with the zeta function, $\zeta(5/4)(\tilde{m})^{\frac{1}{2}}$. Hence, in the $2p_\pi \cdot p_\nu \leq \tilde{m}^2$ region, the series converges. Then the power series rapidly oscillates with $\sqrt{|\delta t|}$ as $S_2 = e^{i\tilde{m}\sqrt{|2v_\nu\sigma_\nu p_\nu \delta t|}\left(1-\frac{p_\pi p_\nu}{\tilde{m}^2}\right)}$. Therefore the present method is valid in the region $2p_\pi \cdot p_\nu \leq \tilde{m}^2$. Outside this region, the power series diverges, and $\Delta_{\pi,\mu}(\delta x)$ has no light-cone singularity. Then $\Delta_{\pi,\mu}(\delta x)$ has only the short-range term.

The integrand of the first term in Eq. (8) is oscillating extremely slowly with δt and its integral of t_1 and t_2 in the finite T

$$i\int_0^{\mathrm{T}} dt_1 dt_2 \frac{\epsilon(\delta t)}{|\delta t|}e^{i\omega_\nu \delta t} = \mathrm{T}(\tilde{g}(\omega_\nu \mathrm{T}) - \pi), \tag{9}$$

slowly approaches constant, while $\tilde{g}(\omega_\nu \mathrm{T})$ satisfies $\frac{\partial}{\partial \mathrm{T}}\tilde{g}(\omega_\nu \mathrm{T})|_{\mathrm{T}=0} = -\omega_\nu$ and $\tilde{g}(\infty) = 0$. The last term in Eq.(9) is cancelled by the short-range term $\tilde{L}_1$ in Eq. (8). Here $\tilde{g}(\omega_\nu \mathrm{T})$ is generated by the superposed waves of the form of light-cone singularity and its effect remains in a macroscopic distance of the order $\frac{2c\hbar E_\nu}{m_\nu^2 c^4}$. We call this the **diffraction** term.

The last term in Eq. (8) is $\frac{2}{\pi}\sqrt{\frac{\sigma_\nu}{\pi}}\int dt_1 dt_2 \tilde{L}_2(\delta t) = \mathrm{T}G_0$, where the constant G_0 is computed numerically. Owing to the rapid oscillation in δt, this integral receives contributions from the microscopic $|\delta t|$ region, and consequently G_0 is constant in T. This term does not depend on σ_ν, and it agrees with the normal probability obtained with the standard method of using plane waves.

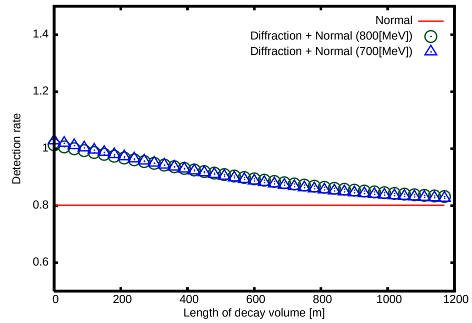

FIG. 1. Total detection rate at a finite distance L is given. The constant shows the normal term, and the diffraction term is written on top of the normal term. The horizontal axis represents the distance in [m], and the normal term is normalized to 0.8. The excess is seen in the distance below 1200m. The neutrino mass, pion energy, and neutrino energy are 1.0 [eV/c^2], 4 [GeV], and 700($\triangle$) and 800($\bigcirc$) [MeV], respectively.

Integrating the neutrino's coordinate $\vec{\mathrm{X}}_\nu$ in Eq. (8), we obtain the momentum-dependent probability. The total volume emerges and is cancelled with V^{-1} of the normalization of the initial pion state. The total probability becomes

$$P = N_2 \int \frac{d^3p_\nu}{(2\pi)^3} \frac{p_\pi \cdot p_\nu}{E_\nu} \left[\tilde{g}(\omega_\nu \mathrm{T}) + G_0 \right], \tag{10}$$

where $N_2 = 8\mathrm{T} g^2 m_\mu^2 \sigma_\nu$ and $\mathrm{L} = c\mathrm{T}$ is the length of decay region. The first term corresponds to $S^{(d)}[\mathrm{T}]$, while the second term corresponds to $S^{(n)}[\mathrm{T}]$. At $\mathrm{T} \to \infty$, the diffraction term vanishes and the probability P agrees with the value of the standard calculation using plane waves. At finite T, the probability has the diffraction component, which is stable under the variation of the pion's momentum.

Next, we evaluate each term of Eq. (10). In the normal term G_0, the energy and momentum are approximately conserved, and the product of the momentums is expressed with the masses $p_\pi \cdot p_\nu = \tilde{m}^2/2$. Integrating the neutrino's angle, we find this term independent of σ_ν, which is consistent with the condition for the stationary state [14]. The transition rate agrees with the value of the ordinary method. However, the diffraction component, $\tilde{g}(\mathrm{T}, \omega_\nu)$, is present in the kinematical region, $|\vec{p}_\nu|(E_\pi - |\vec{p}_\pi|) \leq p_\pi \cdot p_\nu \leq \tilde{m}^2/2$ from the convergence condition and is integrated in this region. This is slightly different from $p_\pi \cdot p_\nu = \tilde{m}^2/2$, hence it is impossible to experimentally distinguish the latter from the former region. We add both terms. The total probability thus obtained is presented in Fig. 1 for the mass of neutrino, $m_\nu = 1\,[\mathrm{eV}/c^2]$, a pion energy $E_\pi = 4\,[\mathrm{GeV}]$, and a neutrino energy $E_\nu = 700$ and

800 [MeV]. For the wave packet size, in which the size of the nucleus having a mass number A, $\sigma_\nu = A^{\frac{2}{3}}/m_\pi^2$ is used. The value becomes $\sigma_\nu = 6.4/m_\pi^2$ for the ^{16}O nucleus and this is used for the evaluation. From this figure, we see that there is an excess of flux at the short distance region L $<$ 600 [m] and the maximal excess is approximately 0.2 of the normal term at L = 0. The slope at the origin L = 0 is determined by ω_ν. The diffraction term is slowly varying with both the distance and energy. The typical length L_0 of this behavior is L_0 [m] $= 2E_\nu \hbar c/m_\nu^2 c^4 = 400 \times E_\nu[\mathrm{GeV}]/m_\nu^2[\mathrm{eV}^2/c^4]$. The neutrino's energy is measured with uncertainty ΔE_ν in the experiments, which is of the order $0.1 \times E_\nu$. This uncertainty is 100 [MeV] for the energy 1 [GeV] and the diffraction components of both energies are almost equivalent to those in Fig. 1. For a larger value of energy uncertainty, the computation is easily made using Eq. (10). Hence the diffraction component is observable if the absolute value of the mass is around 1 [eV/c^2] using the near detector, but it becomes difficult to observe if the mass is less than 0.1 [eV/c^2] using the muon neutrino. In the latter case, an electron neutrino may be used.

The process described with $S[\mathrm{T}]$ has the total probability Eq. (10). In the same experiment, the detection rate of the muon, after neutrinos are integrated, has the same excess. Ordinary experiments of observing the muon, however, does not observe the neutrino and is described by another $S[\mathrm{T}']$, which satisfies the boundary condition for the muon and $\mathrm{T}' = \mathrm{T}_\mu - \mathrm{T}_\pi$ is a time interval for the muon observation. The muon detection probability is computed with a free neutrino, and a probability, then, is expressed in the form of Eq.(10) with $\omega_\nu \rightarrow \omega_\mu = m_\mu^2 c^4/2E_\mu \hbar$. Since the muon is heavy, $\omega_\mu \mathrm{T}'$ becomes very large and $\tilde{g}(\omega_\mu \mathrm{T}')$ at a macroscopic T' vanishes. Thus the detection probability of the muon is not modified, and it agrees with the normal term. The light-cone singularity is formed in both cases, but the diffraction is large in the neutrino and small in the charged lepton.

The detection probability of the muon depends on the boundary condition of the neutrino. When the neutrino is detected at T, the muon spectrum includes the diffraction component, but when the neutrino is not detected, the muon spectrum does not include it. The latter is the standard one, and the former is non-standard, but may be verified experimentally.

Here we compare the neutrino diffraction with the diffraction of light passing through a hole. In the neutrino, the diffraction pattern is formed in a direction parallel to the momentum along with the phase difference $\omega_\nu \delta t$ of the non-stationary wave. Its size is determined by ω_ν, which is extremely small and stable with variations in parameters. Hence,

without fine tuning of the initial energy, the observation is easily made. In the light, the diffraction is formed in a direction perpendicular to the momentum with the phase difference $\omega_{\gamma}^{dB}\delta t$ of the stationary wave. Its shape is determined by ω_{γ}^{dB}, which is large and varies with a change in the parameters. Thus the fine tuning of the initial energy is necessary for the observation.

5 Summary and implications. The neutrino detection probability was computed with the S-matrix of the finite time interval $S[\mathrm{T}]$ and received a large finite size correction in addition to the normal term. The latter agrees with the known value obtained with the standard S-matrix of the plane waves, whereas the former varies with T in the form $C_1\tilde{g}(\omega_\nu \mathrm{T})$, where the constant C_1 depends on the wave packet size. Here $\tilde{g}(\omega_\nu \mathrm{T})$ slowly varies with T and the size of the diffraction pattern is determined by the small angular velocity ω_ν instead of the de Broglie wave length and it is stable under the change in parameters of the initial states. Thus, a neutrino inside this distance reveals the quantum-mechanical diffraction of wave natures and this is not expressed by the asymptotic states of plane waves.

The diffraction gives new corrections to neutrino fluxes but not to those of charged leptons, thus, it is consistent with all previous experiments of charged leptons. The neutrino diffraction in pion decays is a phenomenon distinctive to neutrinos in the non-asymptotic region.

In the case of three masses m_{ν_i} and a mixing matrix $U_{i,\alpha}$, the diffraction term to a neutrino of flavor α is expressed as $\sum_i \tilde{g}(\omega_{\nu_i}\mathrm{T})|U_{i,\alpha}|^2$, whereas the normal term is expressed as $|\sum_i U_{i,\mu}D(i)U_{i,\alpha}^\dagger|^2$ where i is the mass eigenstate, α is the flavor eigenstate, and $D(i)$ is the free wave of m_{ν_i}. Hence the diffraction term depends on the average mass-squared $\bar{m}_\nu^2$, but the normal term depends on mass-squared differences δm_ν^2. At $\mathrm{L} \to \infty$, the diffraction term disappears and the normal terms remain.

The new term has various implications for existing neutrino anomalies and future experiments. One anomaly is an excess of the neutrino flux at near detectors of ground experiments. Fluxes measured by the near detectors of K2K [18] and MiniBooNE [19] show excesses of $10-20$ percent of the Monte Carlo estimations, whereas the excess is not clear in MINOS [20]. These excesses may be connected with the diffraction component. With more statistics, quantitative analysis might become possible to test the diffraction term. Another is the LSND anomaly [21] in which electron neutrinos in pion decays had excesses. Since the diffraction has the origin in the energy-nonconserving term, the helicity suppression does not

work. An electron mode retains in the neutrino flux. Hence the excess in the near-detector regions is attributed to the diffraction component, instead of the flavor oscillation, and the controversy between LSND with others is resolved. [22] Finally, the distance or energy dependence of the neutrino flux may provide a new method for determining the absolute neutrino mass.

Thus the neutrino diffraction appears visible at macroscopic distances and would be confirmed with near detectors. At much larger distances than the above length, the diffraction component disappears and only the normal component including the neutrino flavor oscillation, remains. If the masses do not satisfy $\bar{m}_\nu^2 \gg \delta m_\nu^2$ but satisfy $\bar{m}_\nu^2 \approx \delta m_\nu^2$, then the neutrino fluxes show more complicated behaviors.

A new quantum phenomenon of neutrinos specific to extremely small masses was derived, and its physical quantity determined by the absolute neutrino mass was presented.

In this paper, we ignored the higher-order effects such as the length of pion life time and the pion mean-free-path in studying the quantum effects. We will study these problems and other large-scale physical phenomena of low-energy neutrinos in subsequent papers.

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[1] An S-matrix $S[\mathrm{T}]$ and the Møller operator $\Omega_\pm(\mathrm{T})$ of a finite time interval T satisfies $[S[\mathrm{T}], H_0] = i(\frac{\partial}{\partial \mathrm{T}}\Omega_-(\mathrm{T}))^\dagger\Omega_+(\mathrm{T}) - i\Omega_-(\mathrm{T})^\dagger\frac{\partial}{\partial \mathrm{T}}\Omega_+(\mathrm{T})$ with a free Hamiltonian. Hence, the energy is not conserved by $S[\mathrm{T}]$, and the final states of non-conserved energies contribute.

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